Quantitative Conclusions from Heavy-Ion Collisions

Scott Pratt, Michigan State University

MADAI Collaboration

Models and Data Analysis Initiative

http://madai.us

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY · Duke UNIVERSITY · THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL · renci

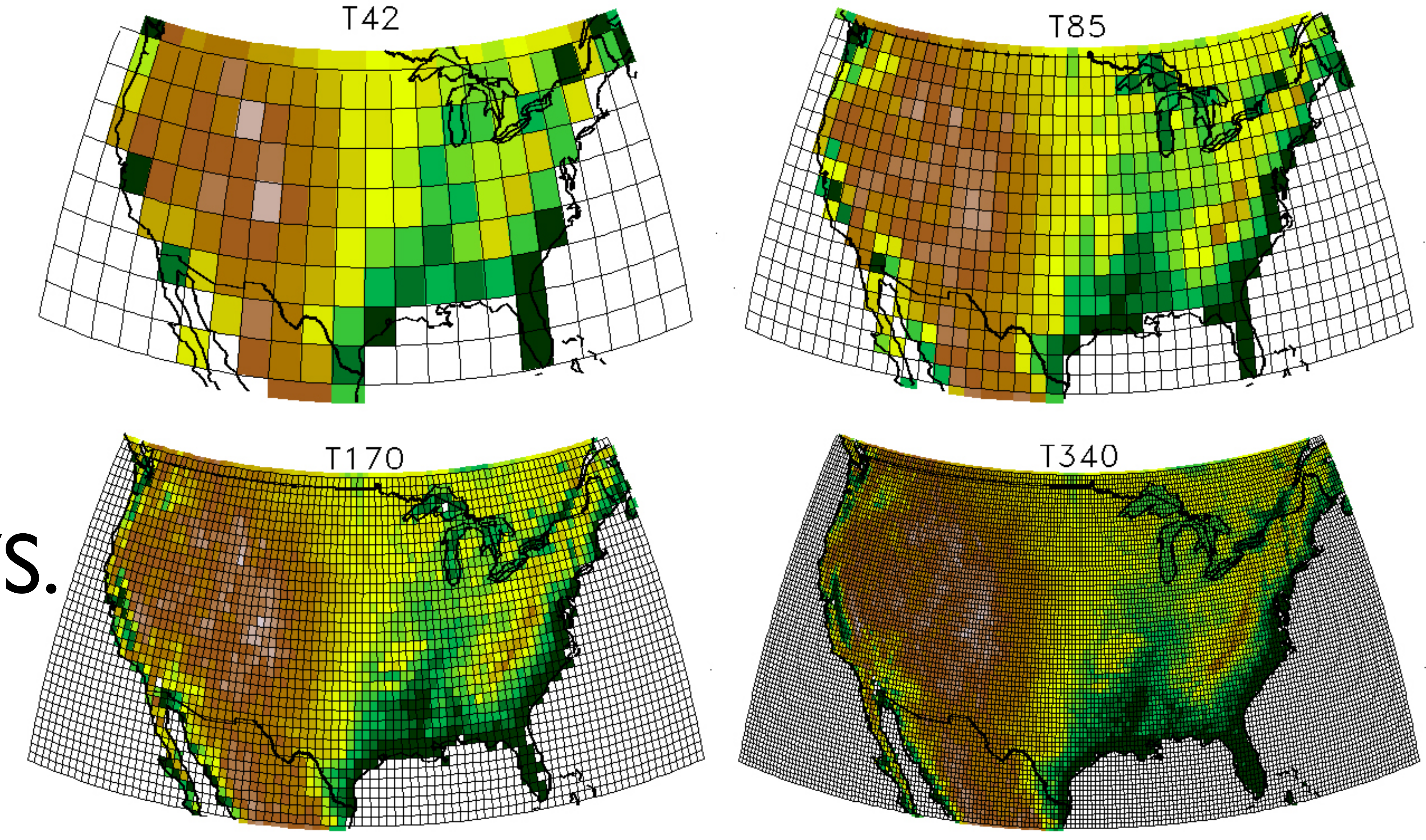
1st MADAI Collaboration Meeting, SANDIA 2010

Common Challenge

BIG Data

Large Heterogenous Data Sets

VS.

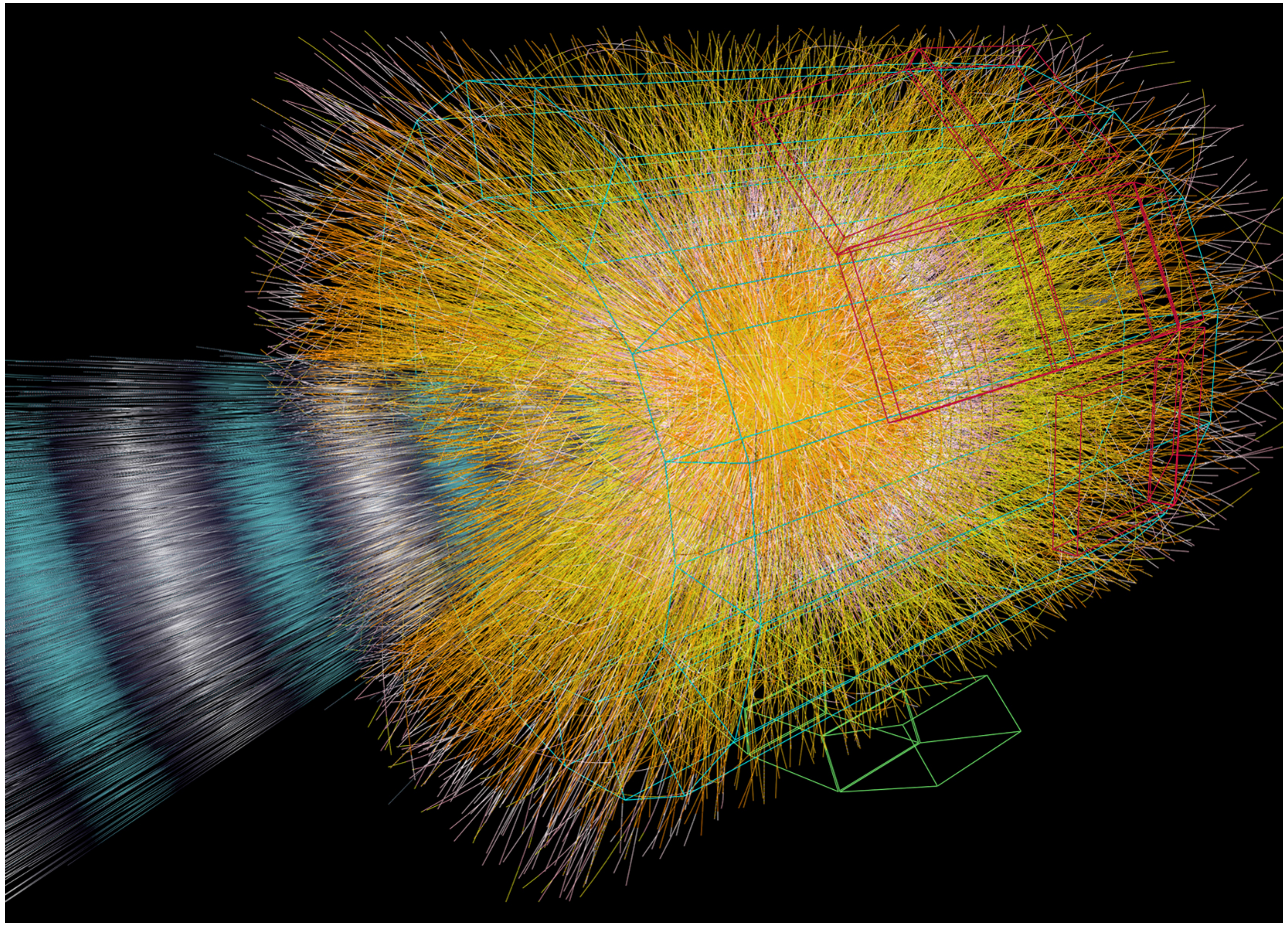


BIG Models
Many parameters
Numerically Intensive

An Example: Relativistic Heavy Ion Physics

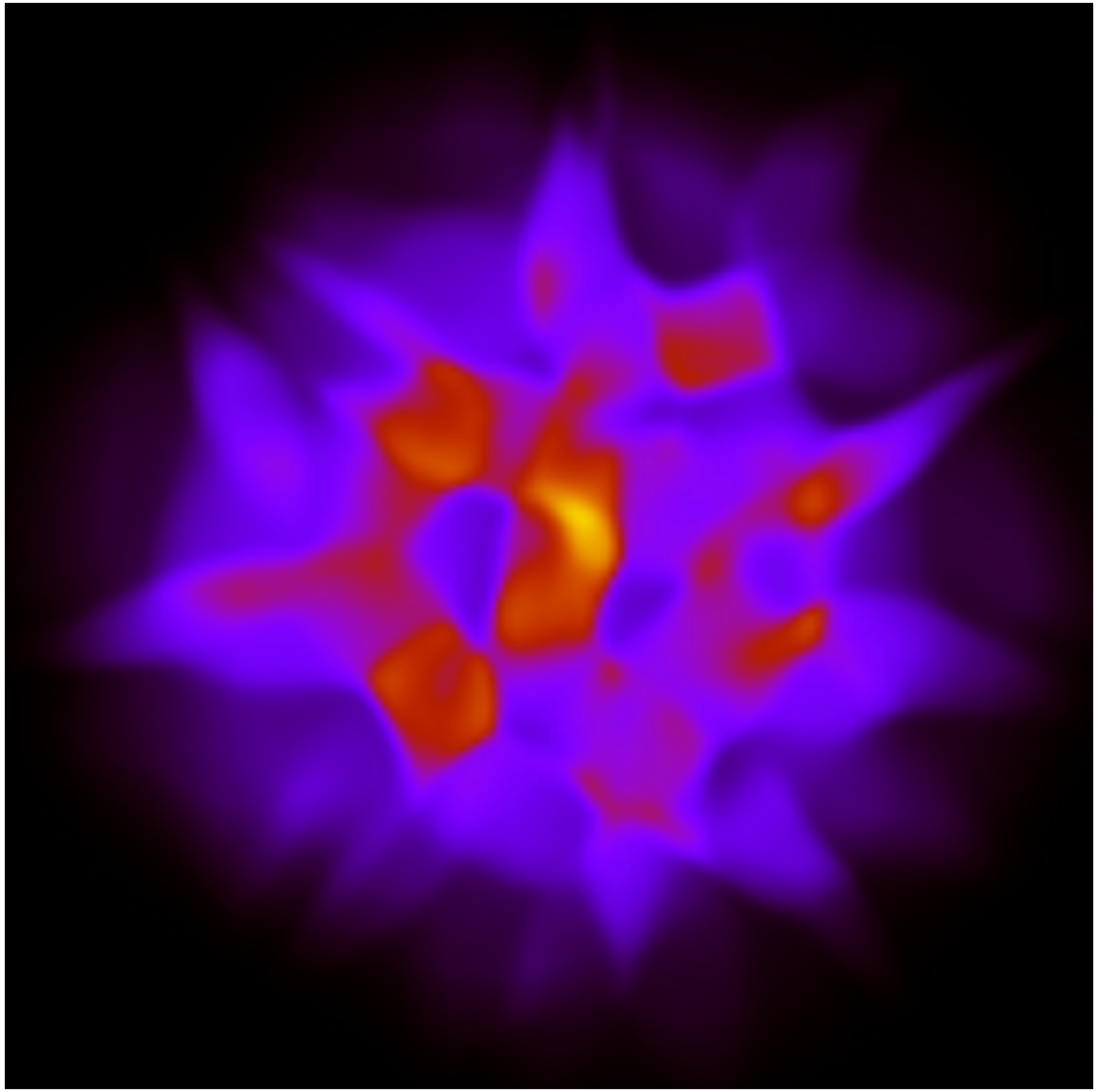
Collisions of Au&Au, Pb+Pb…
at RHIC(BNL) or LHC(CERN)

Numerous Classes of Observables



Goal: Determine properties of super-hadronic matter (**Q**uark-**G**luon **P**lasma)

An Example: Relativistic Heavy Ion Physics

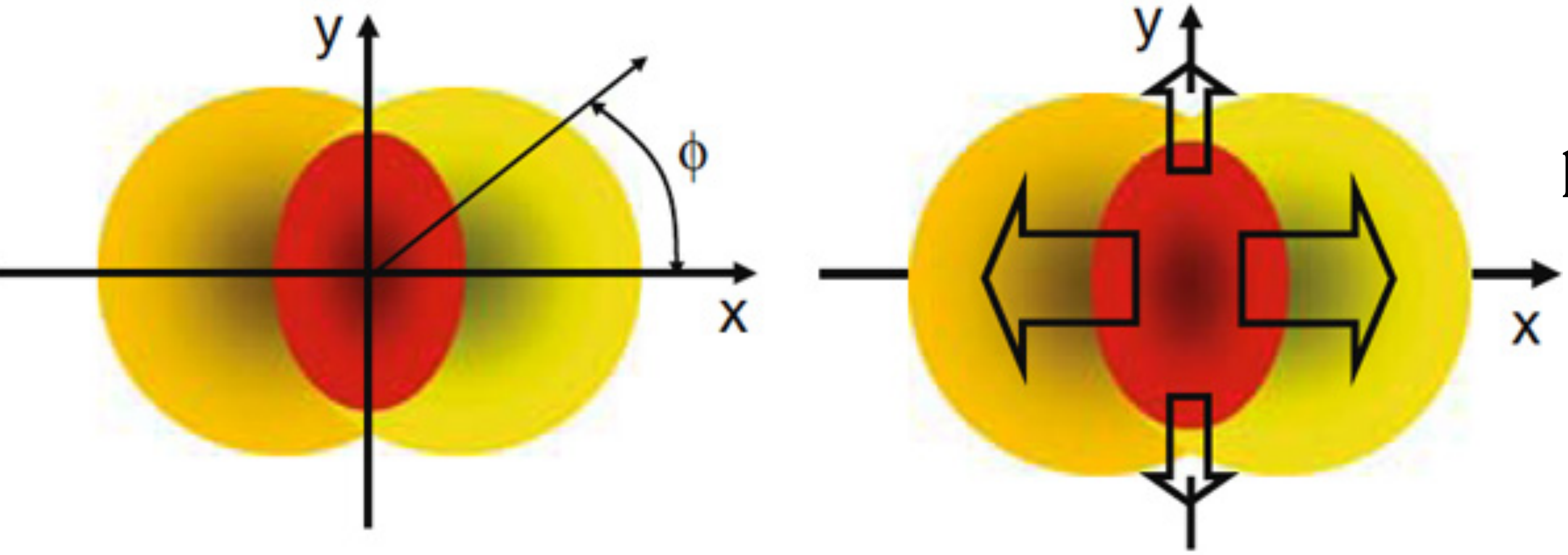


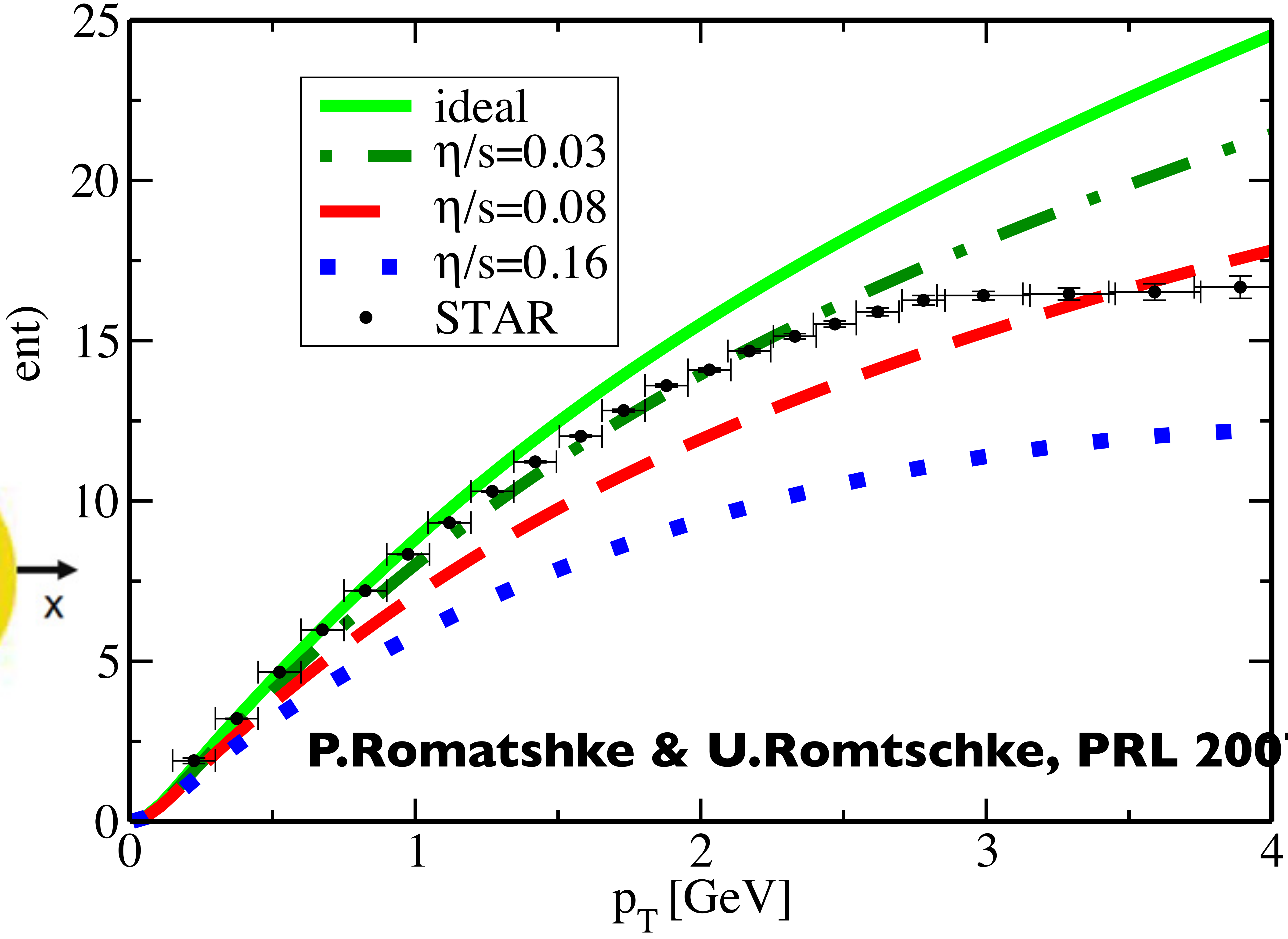
MODEL COMPONENTS

- Thermalization (First fm/c)
- Viscous Hydrodynamics (First ~5-10 fm/c)
- Hadron Simulation (Dissolution & Breakup)
- Numerous parameters (up to few dozen)
- ~Days of CPU to study one point in parameter space

How this was done before ($v2$ and η/s)

$$v_2 \equiv \langle \cos 2\phi \rangle$$





P.Romatshke & U.Romtschke, PRL 2007

PROBLEM

v2 depends on ....

- viscosity
- saturation model
- pre-thermal flow
- Eq. of State
- T-dependence of η/s
- initial T_{xx}/T_{zz}
- . . . .

e.g. Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

Correct Way (MCMC)

- Simultaneously vary *N* model parameters $\boldsymbol{x_i}$
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})})^2}{2\sigma_a^2}\right\}$$

- Use all observables $\boldsymbol{y_a}$
- Obtain representative sample of posterior

Difficult Because...

I. Too Many Model Runs

Requires running model ~10^6 times

II. Many Observables

Could be hundreds of plots,
each with dozens of points
Complicated Error Matrices

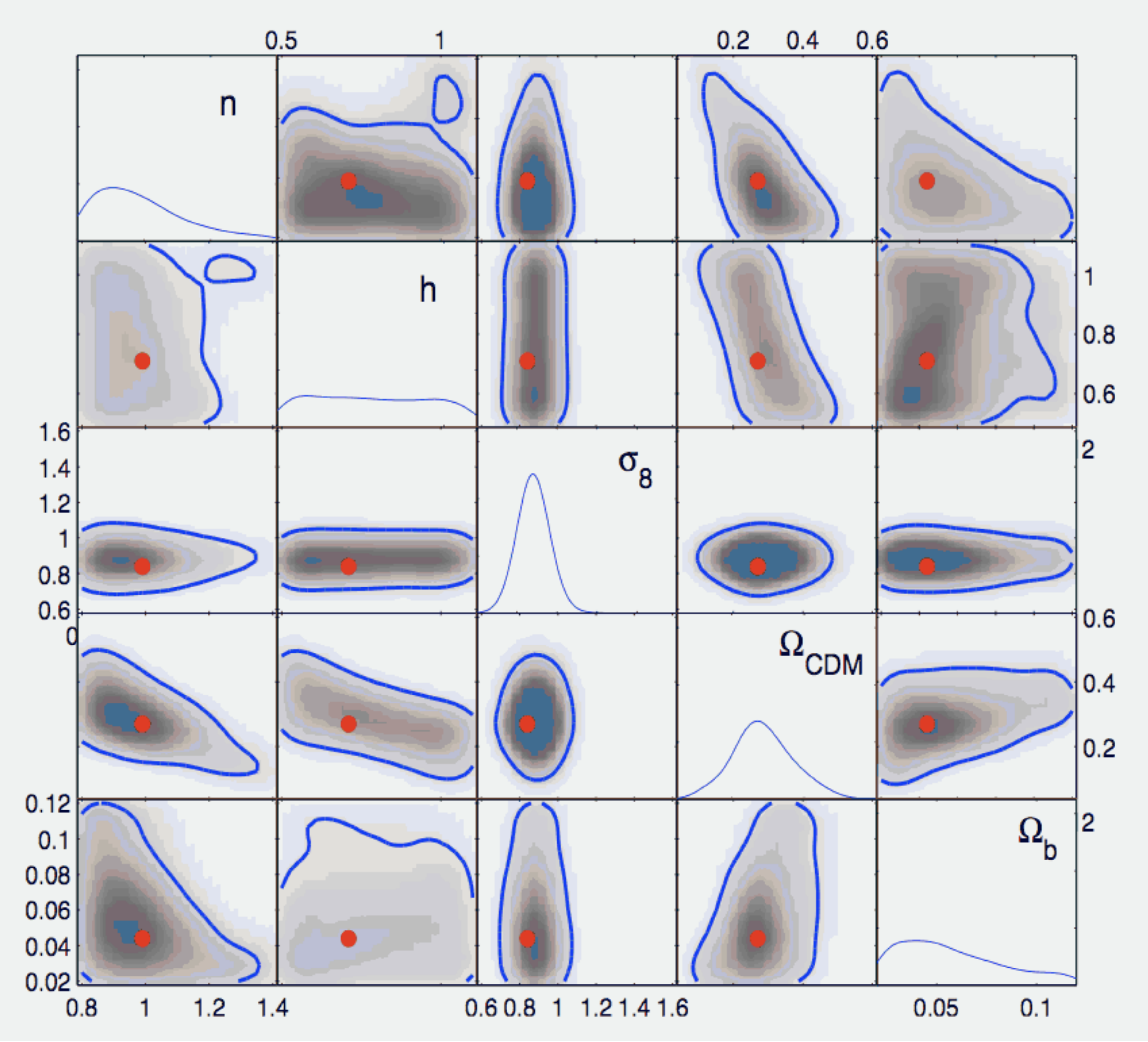
Model Emulators

1. **Run the model ~1000 times**
   Semi-random points (LHS sampling)

2. **Determine Principal Components**
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

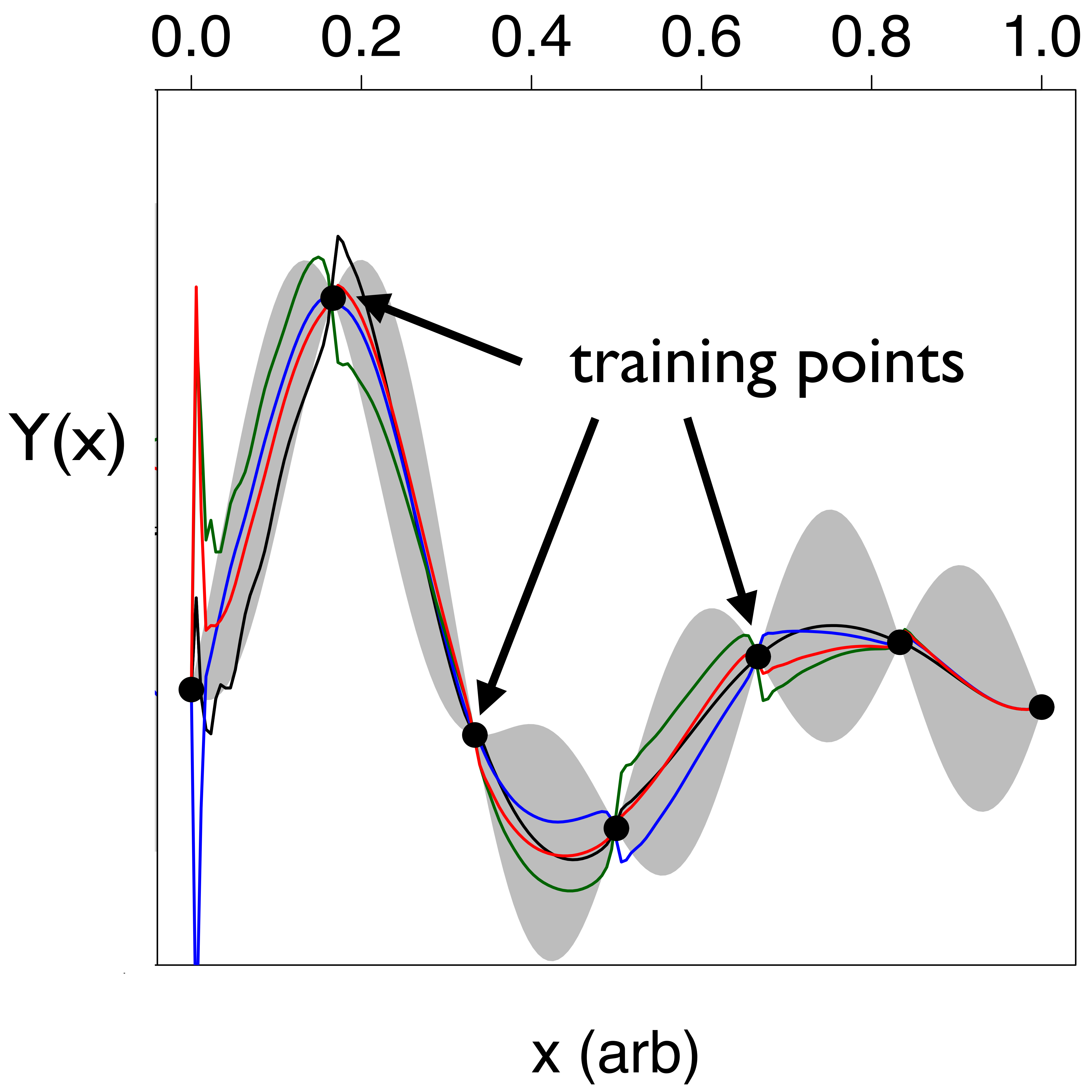
3. **Emulate z_a (Interpolate) for MCMC**
   Gaussian Process…

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{ -\frac{1}{2} \sum_a (z_a^{(\text{emulator})}(\mathbf{x}) - z_a^{(\text{exp})})^2 \right\}$$



S. Habib, K. Heitman, D. Higdon, C. Nakhleh & B. Williams, PRD(2007)

Emulator



- Gaussian Process
  - Reproduces training points
  - Assumes localized Gaussian covariance
  - Must be trained, i.e. find “hyper parameters”
- Other methods also work

14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- π, K, p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Size
- v_2 Weighted by p_t

Initial State Parameters

$$\epsilon(\tau = 0.8\text{fm}/c) = f_{\text{wn}}\epsilon_{\text{wn}} + (1 - f_{\text{wn}})\epsilon_{\text{cgc}},$$

$$\epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2\sigma_{\text{sat}}} \{1 - \exp(-\sigma_{\text{sat}} T_B)\} + (A \leftrightarrow B)$$

$$\epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \{1 - \exp(-\sigma_{\text{sat}} T_{\text{max}})\}$$

$$T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\text{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

Equation of State and

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

2 parameters for EoS, 2 for η/s

DATA

1. **Experiments reduce PBs to 100s of plots**

2. **Choose which data to analyze**
   Does physics *factorize*?

3. **Reduce plots to a few representative numbers, y_a**

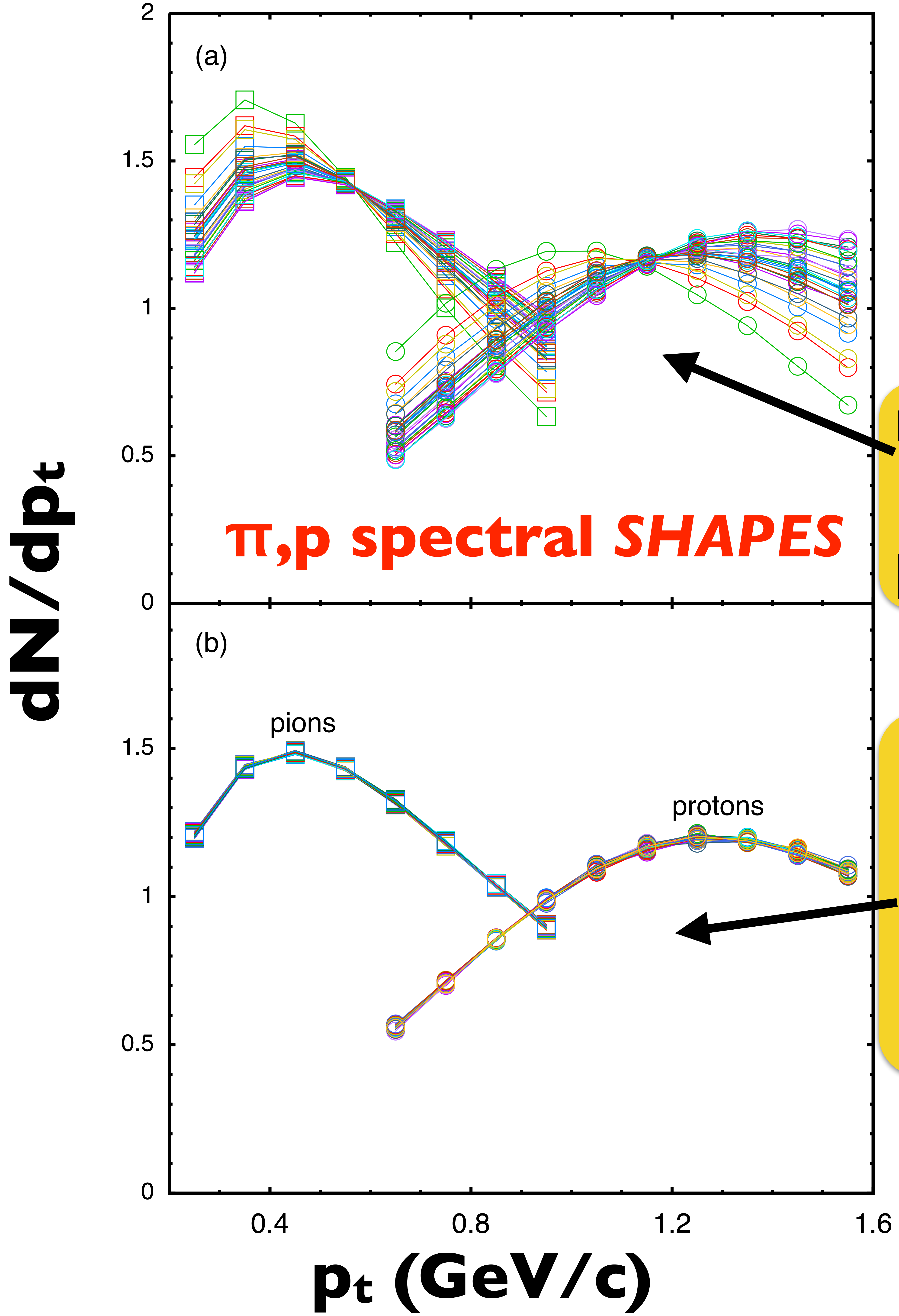
4. **Transform to principal components, z_a**

$$\mathcal{L} \sim \exp\left\{ \frac{-1}{2} \sum_a \left(z_a - z_a^{(\mathrm{exp})}\right)^2 \right\}$$

5. **Resolving power of RHIC/LHC data reduced to ≲10 numbers!**

Checking the Distillation

Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$



model spectra from 30 random points in parameter prior

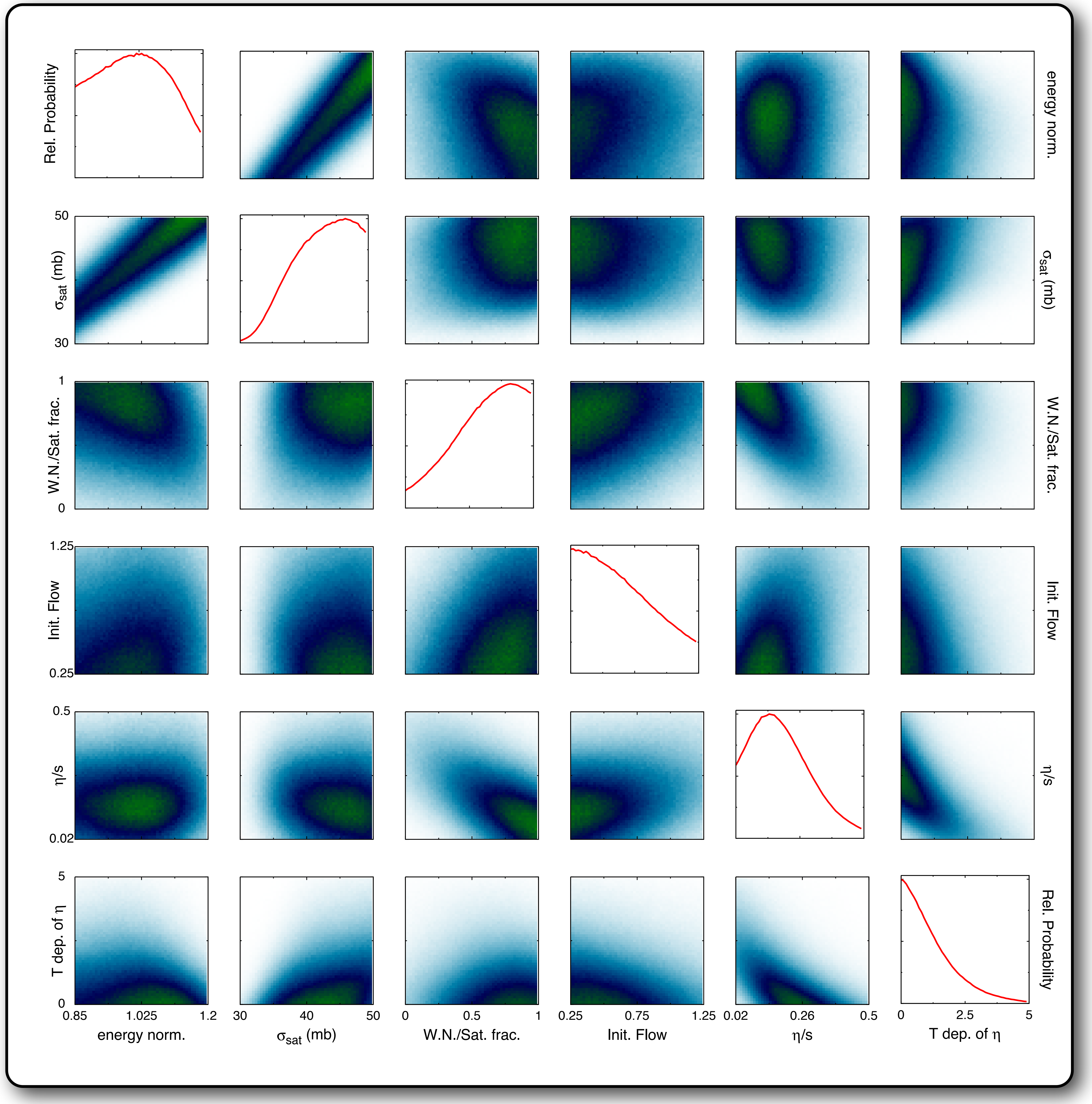
74 pion spectra: with 573<$\langle p_t \rangle_\pi$< 575 MeV

44 proton spectra: with 1150<$\langle p_t \rangle_p$< 1152 MeV

Two Calculations

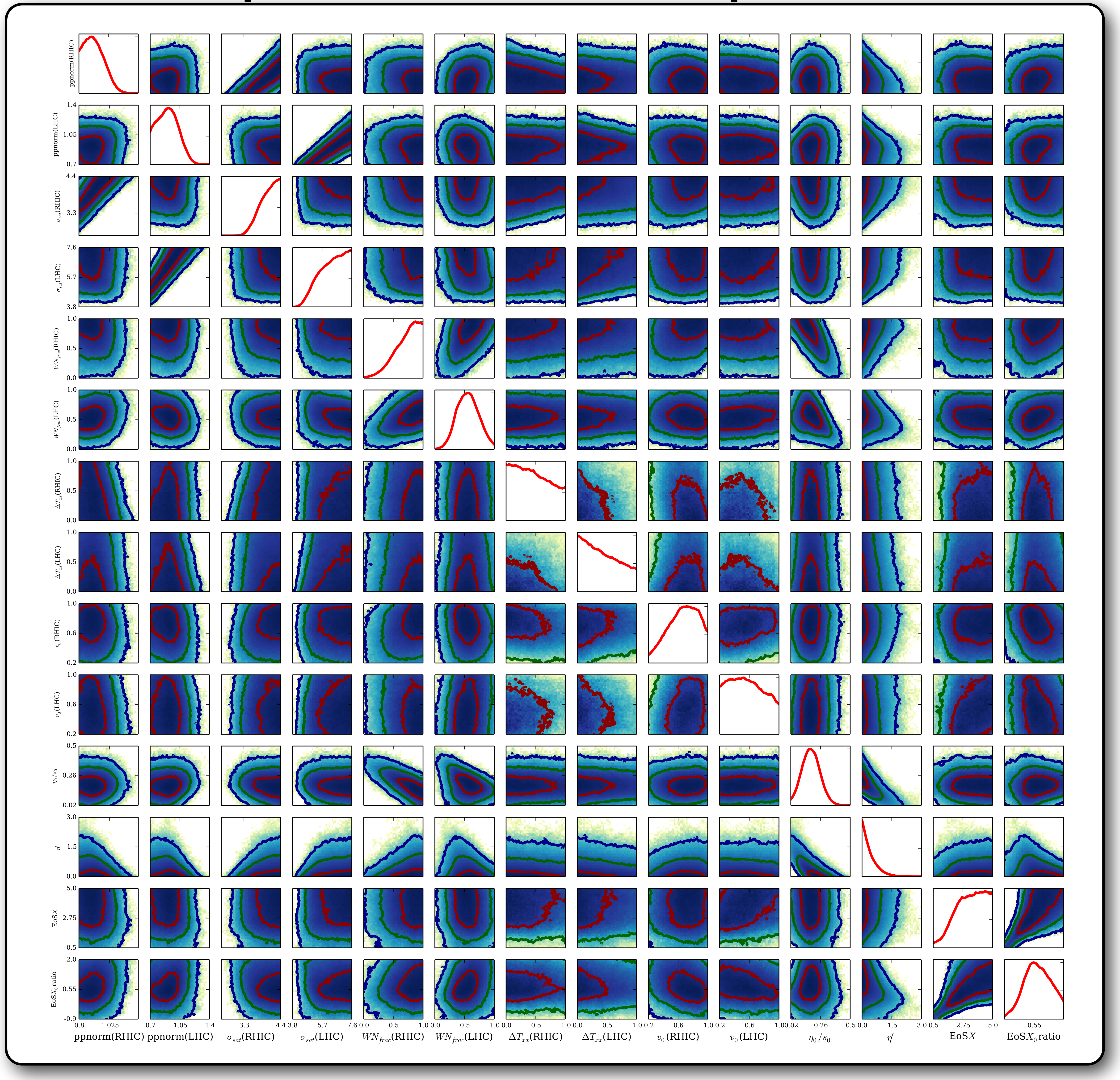
J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, PRC 2014

RHIC Au+Au Data
6 parameters

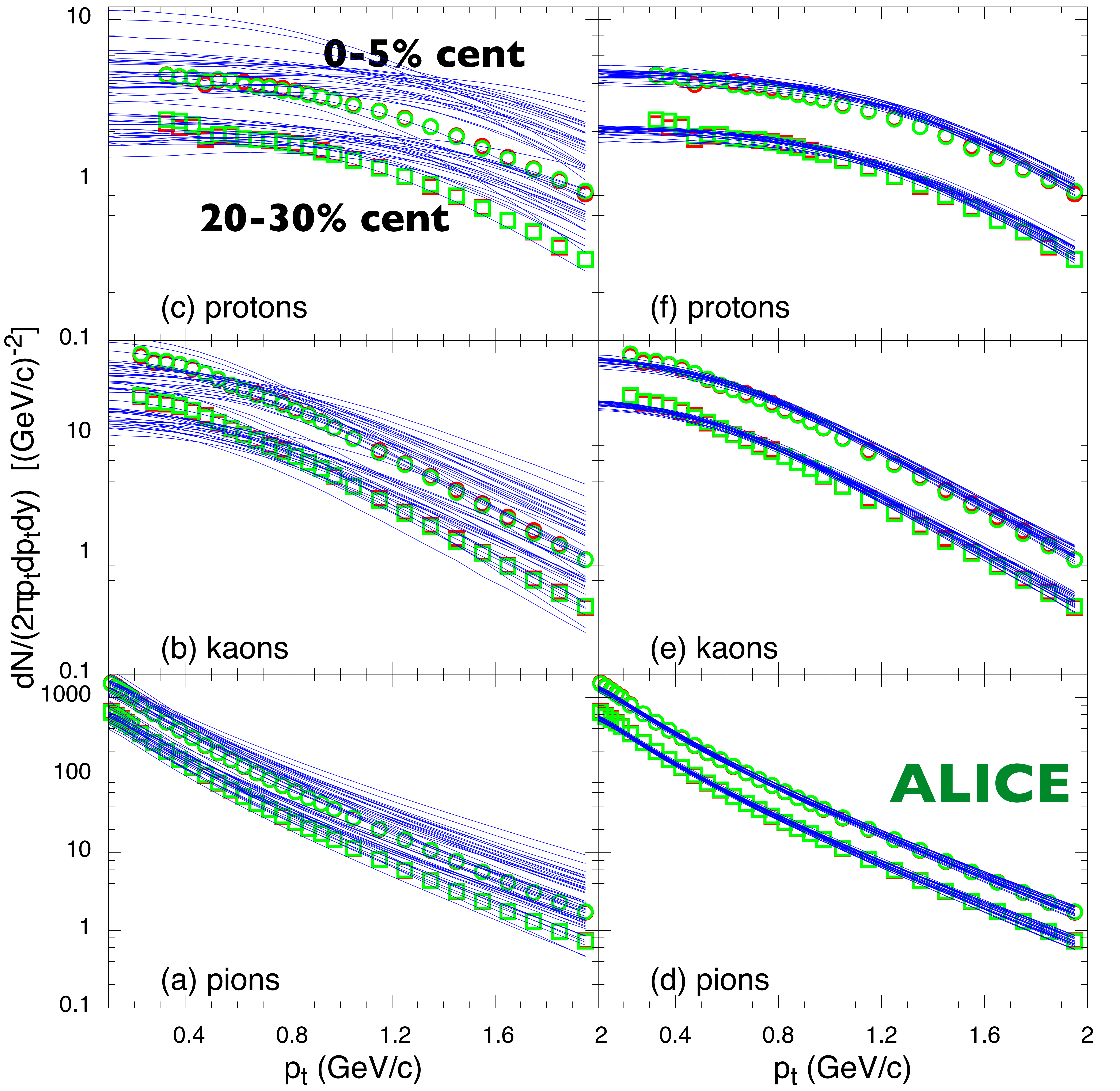


S.P., E.Sangaline, P.Sorensen & H.Wang, PRL 2015

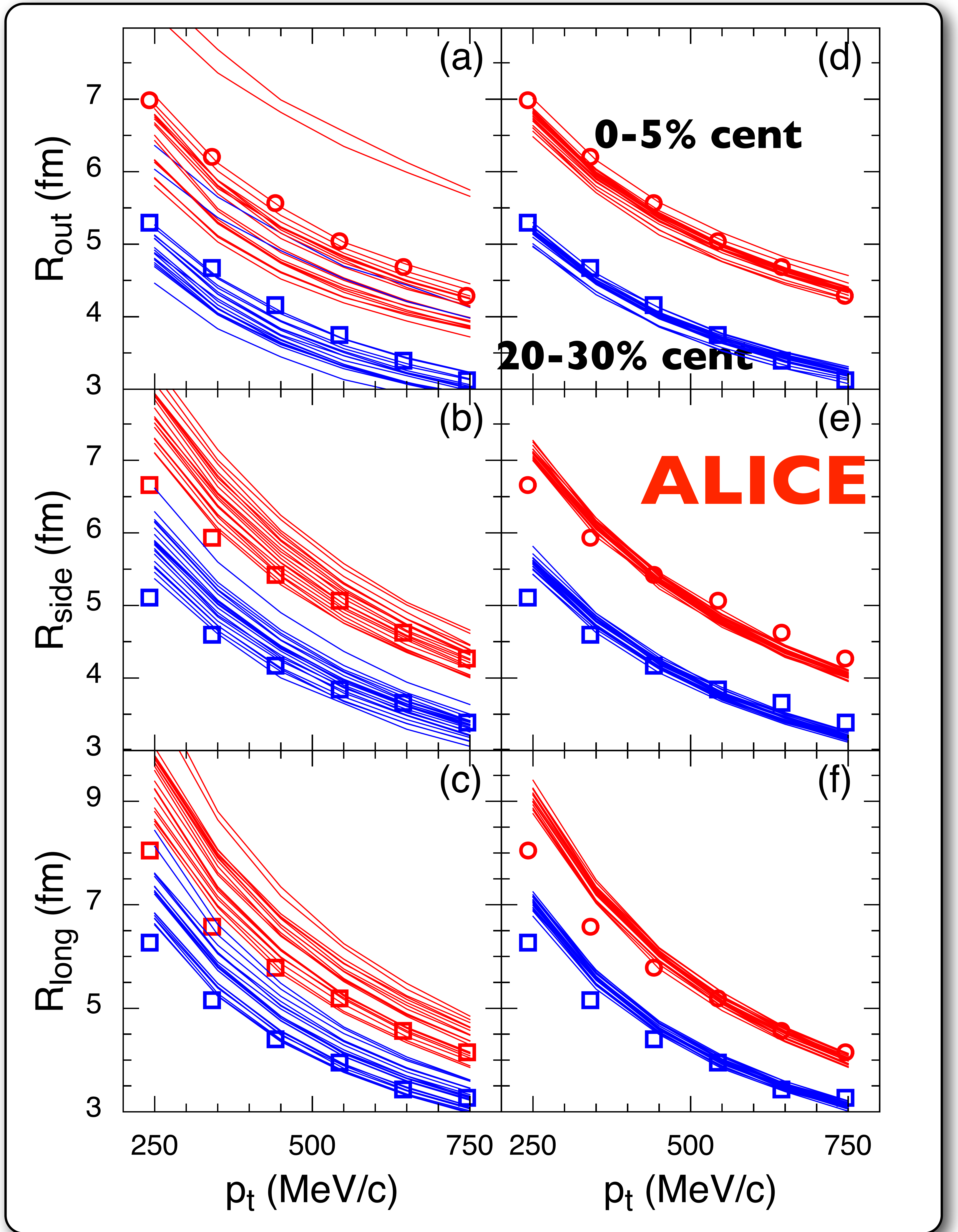
RHIC Au+Au and LHC Pb+Pb Data
14 parameters, include Eq. of State



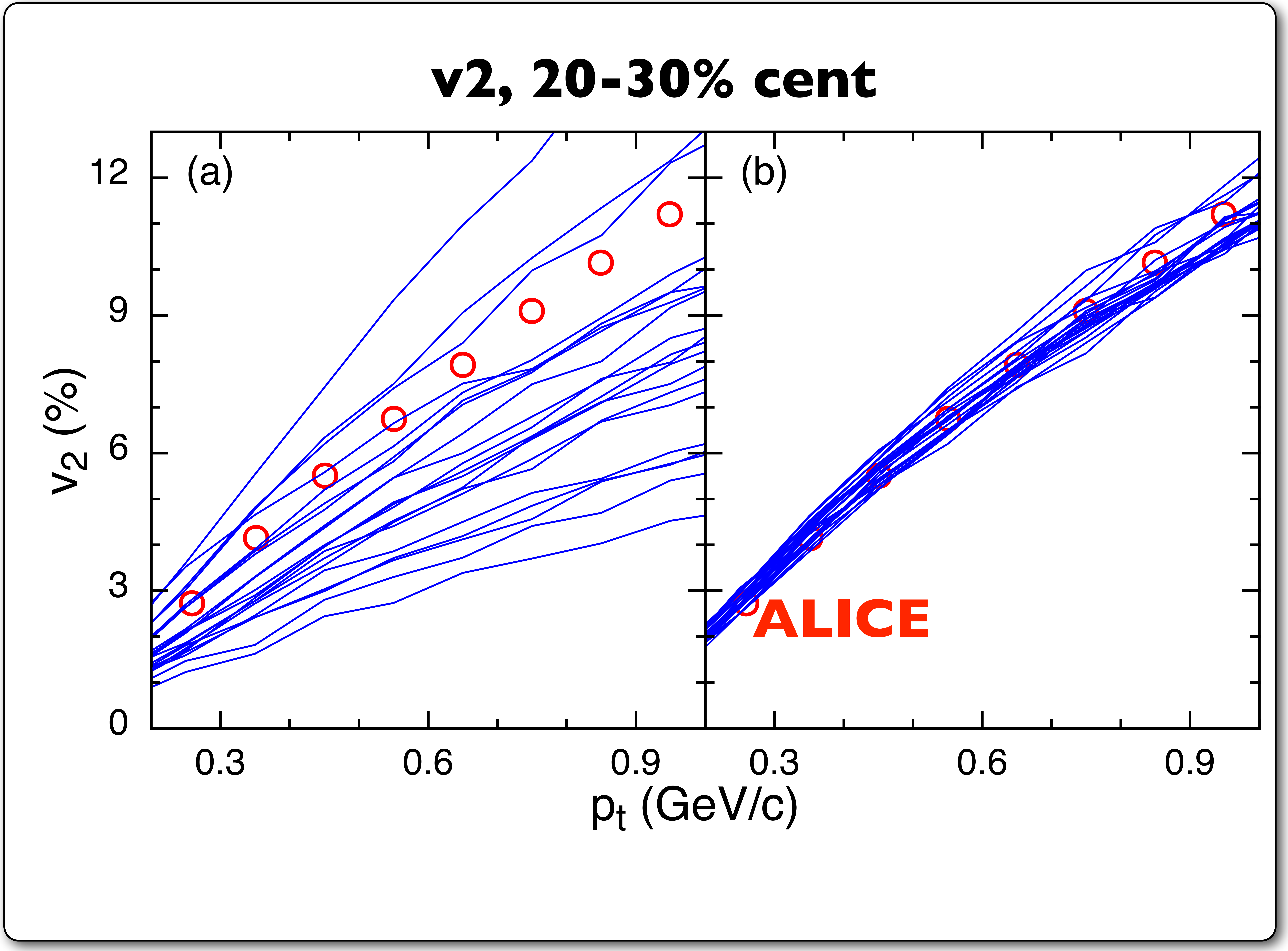
Sample Spectra from Prior and Posterior



Sample HBT from Prior and Posterior



Sample V2 from Prior and Posterior

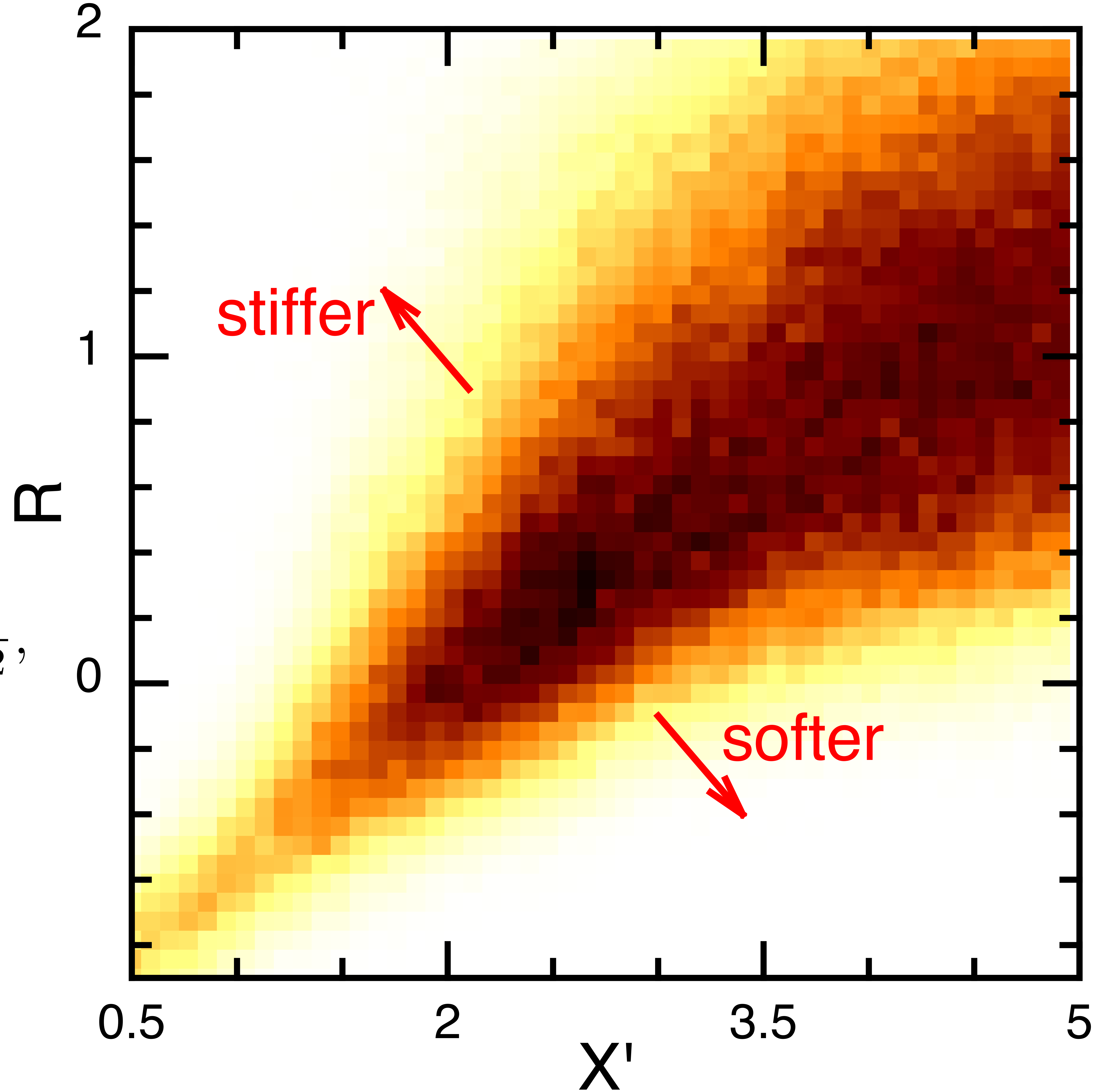


Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right)\frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

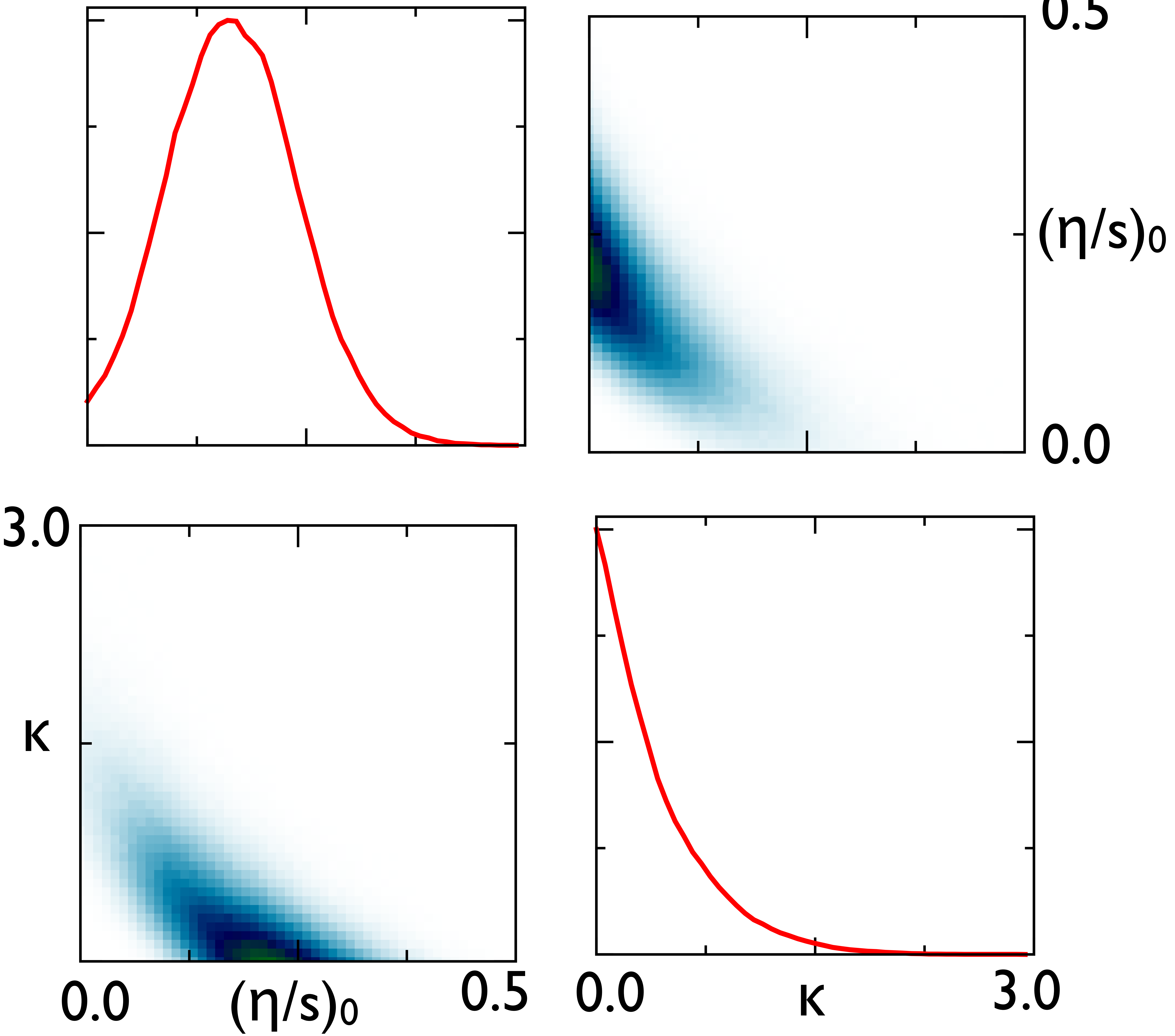
$$x \equiv \ln \epsilon/\epsilon_h$$



Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
c_s^2 (speed of sound squared)
0.3
0.2
0.1
UNCONSTRAINED
CONSTRAINED
BY DATA
HADRON GAS
150
200
250
300
350
T (MeV)

η/s(T)

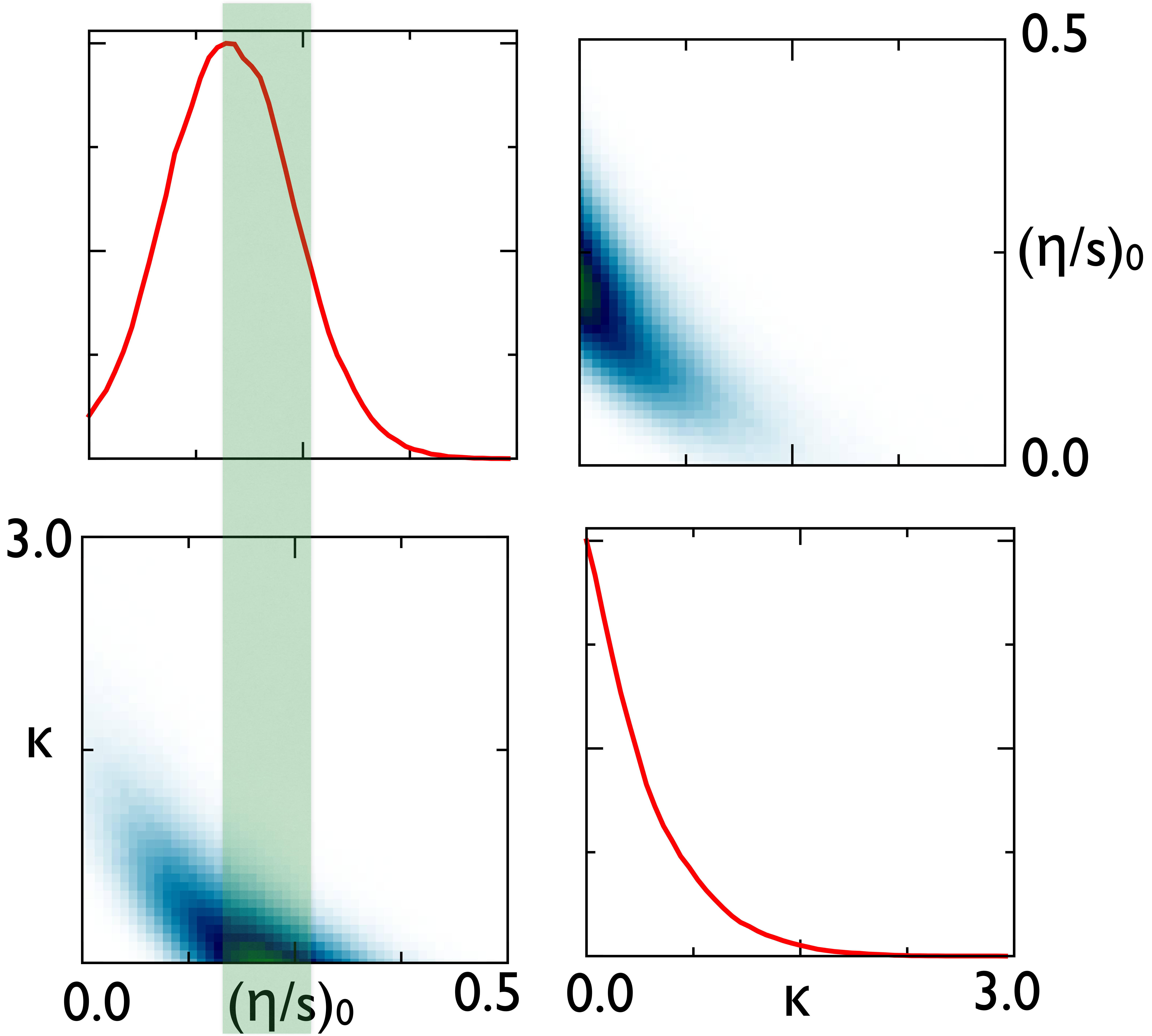
$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$



What should you expect for η/s at T=165 MeV?

• ADS/CFT:	0.08
• Perturbative QCD:	> 0.5 ($\sigma \approx$ 3 mb)
• Hadron Gas:	$\approx$ 0.2 ($\sigma \approx$ 30 mb)

Extracted η/s at T=165 MeV consistent with expectations for hadron gas!



How does changing $y_{a,\mathrm{exp}}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

We need

$$\frac{\partial}{\partial y_a^{(\mathrm{exp})}} \langle\langle x_i \rangle\rangle$$

NOT

$$\frac{\partial}{\partial x_i} y_a^{(\mathrm{mod})}$$

E.Sangaline and S.P., arXiv 2015

How does changing $y_{a,exp}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

$$\langle\langle x_i \rangle\rangle = \frac{\langle x_i \mathcal{L} \rangle}{\langle \mathcal{L} \rangle}$$

$$\frac{\partial}{\partial y_a^{(\mathrm{exp})}} \langle\langle x_i \rangle\rangle = \langle\langle x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle - \langle\langle x_i \rangle\rangle \langle\langle (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle$$

$$= \langle\langle \delta x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle$$

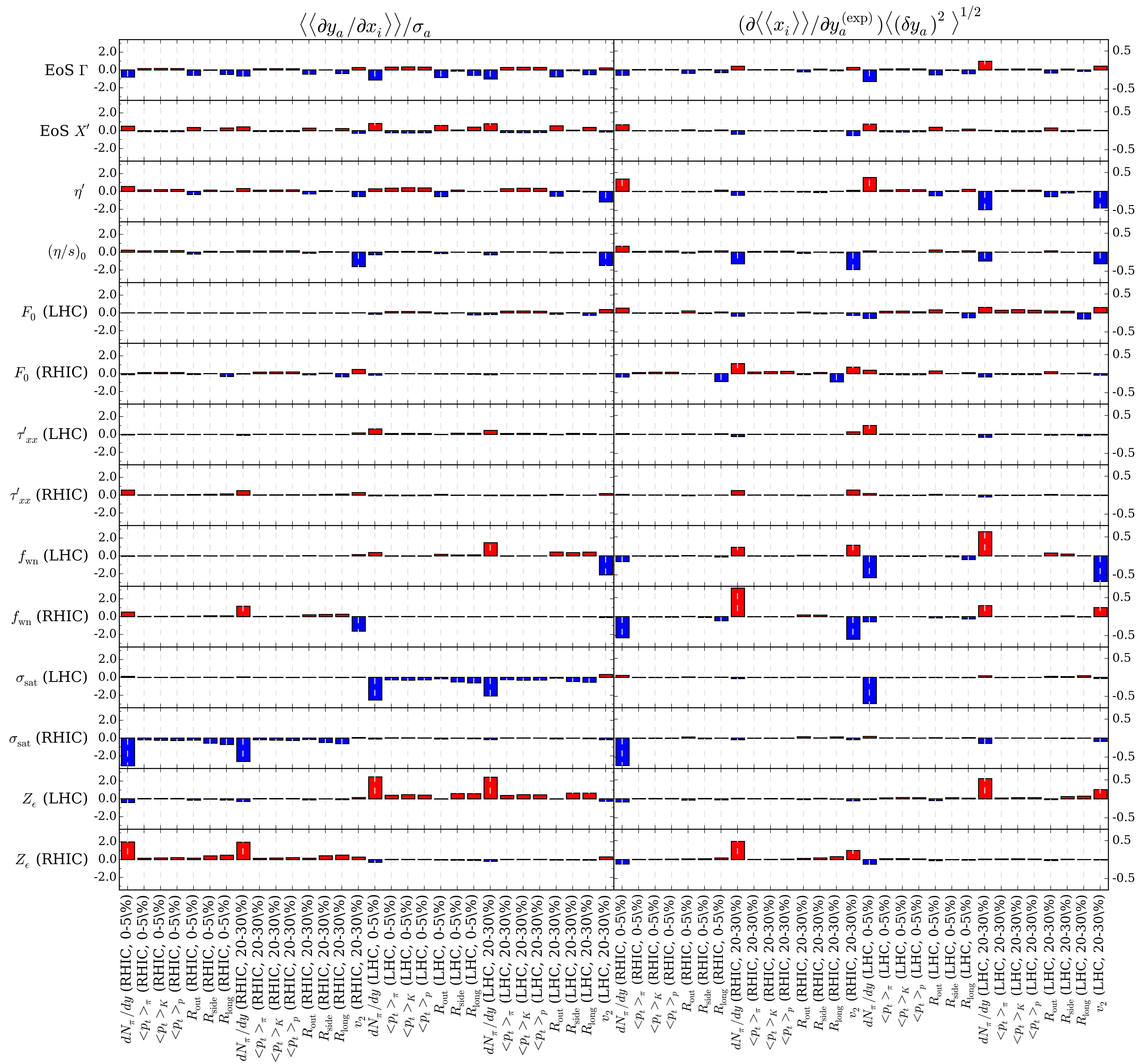
$$= -\Sigma_{ab}^{-1} \langle\langle \delta x_i \delta y_b \rangle\rangle \qquad \text{(for Gaussian)}$$

$$\delta x_i = x_i - \langle\langle x_i \rangle\rangle, \quad \delta y_a = y_a - y_a^{(\mathrm{exp})}$$

can find similar relation for $\frac{\partial}{\partial \sigma_a} \langle\langle \delta x_i \delta x_j \rangle\rangle$

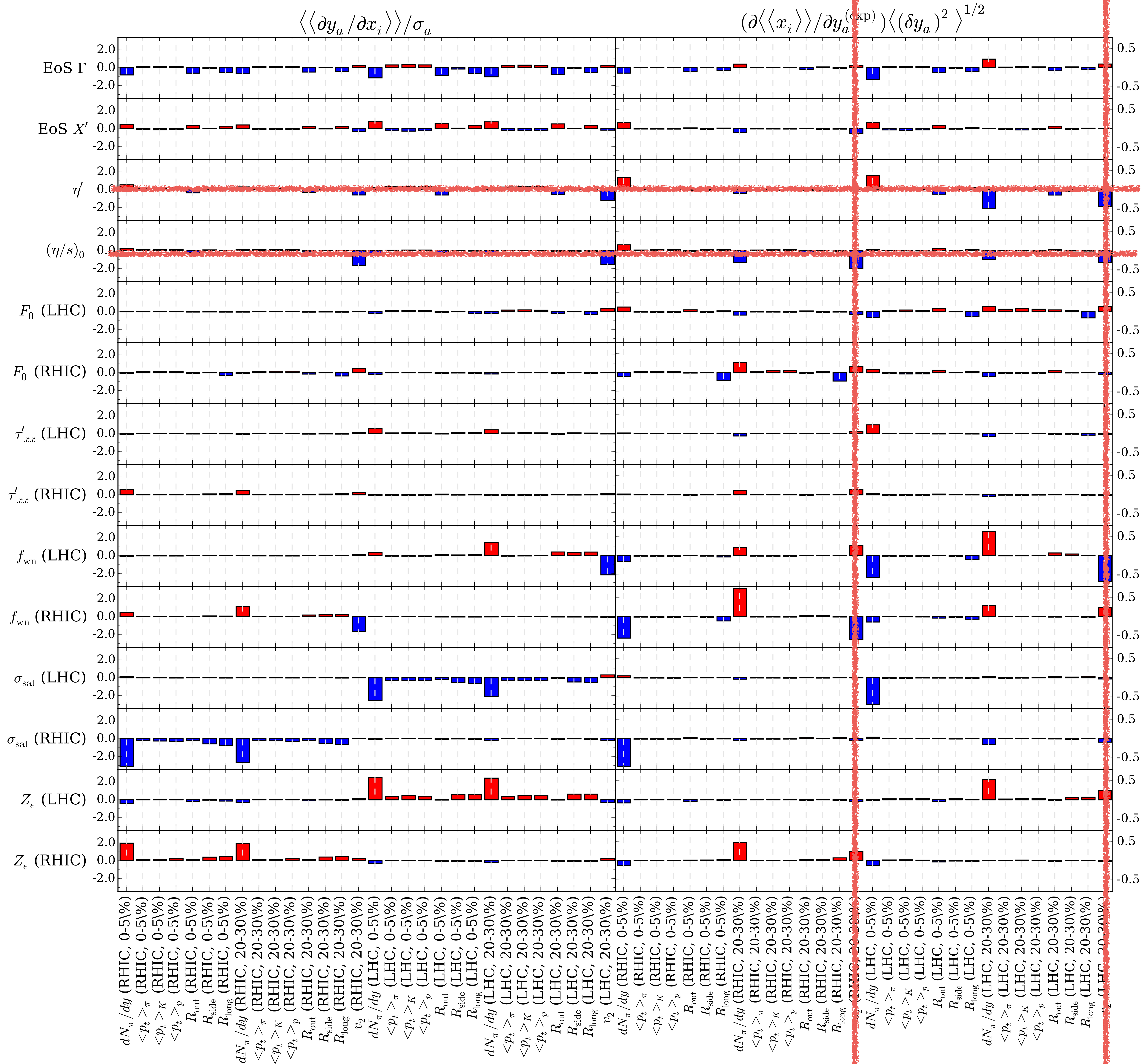
E.Sangaline and S.P., arXiv 2015

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$



$$\langle \delta y_a \delta y_a\rangle^{1/2}\,\frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$

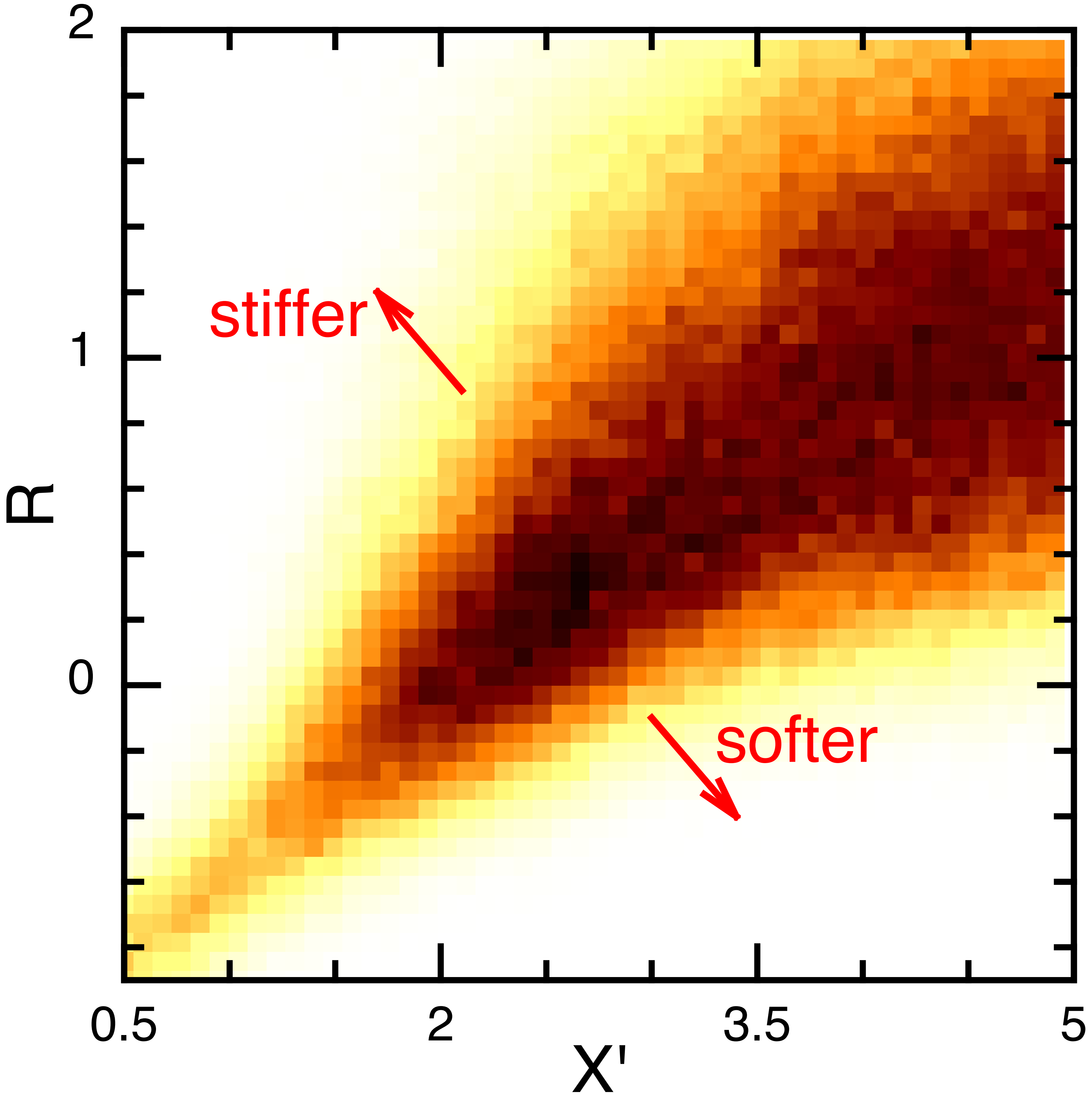


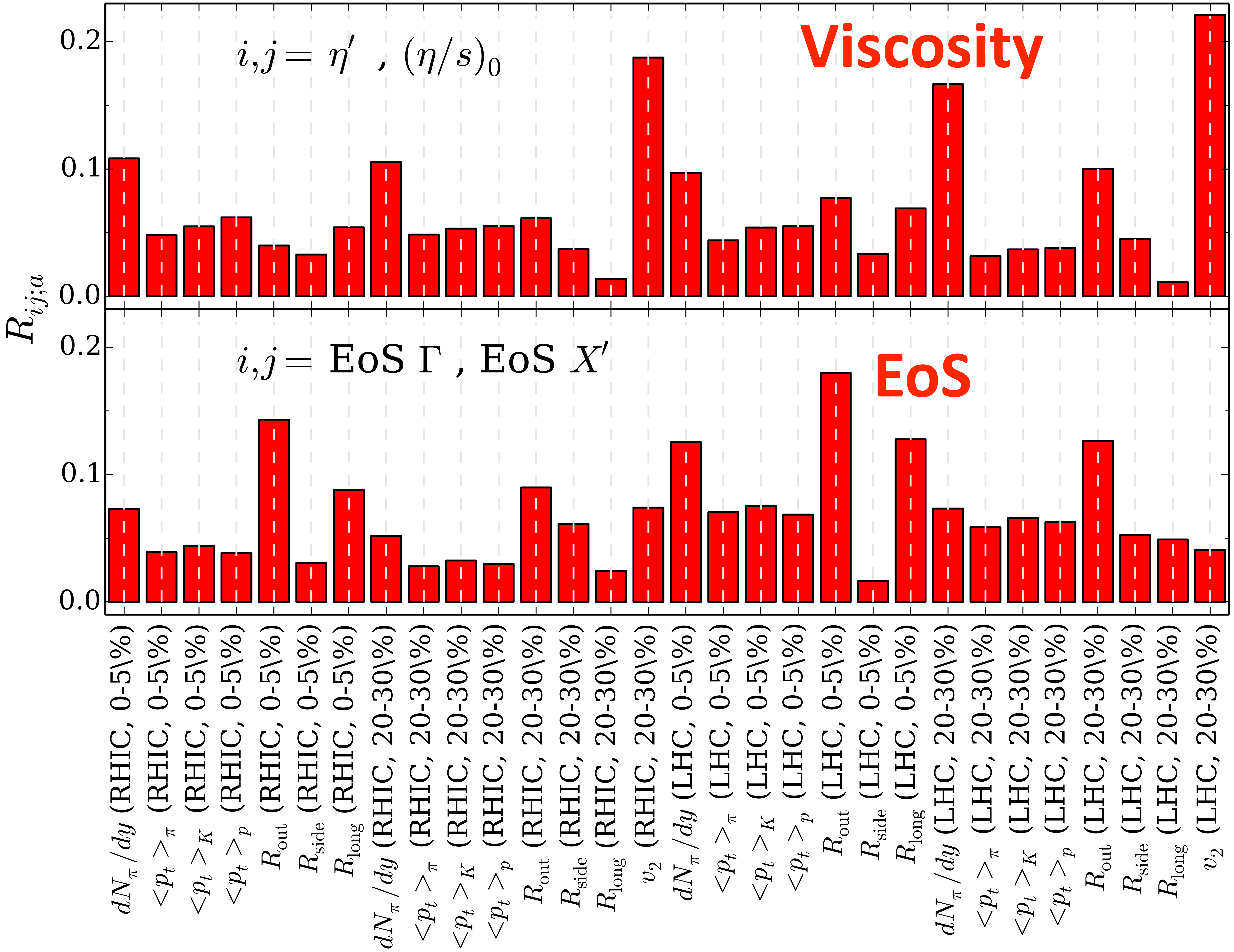
$$\langle \delta y_a \delta y_a \rangle^{1/2}\,\frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

0.5
-0.5
η'
η0
v2(RHIC)
v2(LHC)
$\langle \delta y_a \delta y_a \rangle^{1/2} \left. \frac{\partial x_i}{\partial y_a} \right|_{y_{b \neq a}}$

2-Parameter Sensitivity

$$\frac{d}{d\sigma_y}\sqrt{\begin{vmatrix}\langle\langle\delta x_1\delta x_1\rangle\rangle & \langle\langle\delta x_1\delta x_2\rangle\rangle\\ \langle\langle\delta x_1\delta x_2\rangle\rangle & \langle\langle\delta x_2\delta x_2\rangle\rangle\end{vmatrix}}\,\langle\delta y\delta y\rangle^{1/2}$$





What determines viscosity?

- **Both v_2 and multiplicities**
- **T-dependence comes from LHC v_2**

What determines EoS?

- Lots of observables
- Femtoscopic radii are important

CONCLUSIONS

- Robust
- Emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from RHIC & LHC data
- Other parameters not as well constrained
- Heavy-Ion Physics can be a Quantitative Science!!!!